

TOWARDS DATA-DRIVEN BOILING MODELS USING A MIXTURE-OF-EXPERTS APPROACH

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ABSTRACT

Accurate prediction of boiling heat transfer remains a major challenge due to the complex, multi-scale interactions underlying nucleate boiling phenomena. Existing models based on heat flux partitioning (HFP) often rely on empirical or semi-empirical sub-models with limited validity across different flow and thermal conditions. In this work, we propose a data-driven mixture-of-experts (MoE) framework that integrates multiple semi-mechanistic and semi-empirical HFP models through a neural gating network trained on subcooled flow boiling data. Each expert represents a distinct combination of HFP formulation and bubble dynamics sub-models, while the gating function adaptively weights their contributions based on non-dimensional flow and thermophysical features. The approach is trained and tested on a comprehensive experimental database covering a range of pressures, subcoolings, and velocities. Results show that the MoE achieves improved global predictive accuracy compared to individual boiling models, with mean absolute percentage errors of 4.9%, 19.3%, and 23.0% across training, interpolation-testing, and extrapolation-testing datasets, respectively. Feature ablation analysis reveals that the model's weighting behaviour is most sensitive to pressure and subcooling. The proposed framework demonstrates a promising path towards adaptive, data-assisted boiling models which have the potential to retain the interpretability of existing formulations while improving their applicability across operating regimes.

KEYWORDS: Boiling models, sub-cooled flow boiling, heat flux partitioning, machine learning, mixture-of-experts

1. INTRODUCTION

Accurate modelling of nucleate boiling is essential to the design and safety analysis of a wide range of thermal systems, including nuclear reactors, where boiling plays a dominant role in core heat removal. In nuclear thermal hydraulics, subcooled and saturated boiling processes set the heat transfer capability of fuel assemblies and determine the available margin to departure from nucleate boiling (DNB) and the critical heat flux (CHF). DNB refers to the point at which the liquid can no longer re-wet the heated surface, leading to the formation of persistent dry areas, while CHF denotes the maximum heat flux that can be sustained before this transition occurs. Crossing this limit triggers a sharp reduction in cooling performance, often termed the boiling crisis, and therefore defines a primary safety constraint for reactor operation. Despite decades of research, predicting heat transfer in the nucleate-boiling regime remains an open challenge due to the complex, multiscale physics involved [1]. Existing models often rely on simplified assumptions or empirically tuned correlations with limited ranges of validity, reducing reliability outside narrow calibration regimes. The development of predictive, generalisable boiling models that can be trusted across operating conditions is therefore a critical need for advancing simulation capabilities in nuclear and other safety-critical applications.

In multiphase computational fluid dynamics for cases of industrial interest such as reactor thermal hydraulics, wall boiling is typically modelled within the Eulerian–Eulerian (E-E) two-fluid framework due to the intractable cost of alternative CFD methods such as volume-of-fluid or Lagrangian particle-based formulations. In the Eulerian-Eulerian approach, the liquid and vapour phases are treated as interpenetrating continua and their

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transport equations are closed through a variety of interfacial and turbulence models [2]. As the microscale details of bubble nucleation, growth, and departure are not resolved in such simulations, additional wall-boiling models are required to represent the heat transfer from the heated surface. A widely adopted approach is the heat flux partitioning (HFP) framework, which decomposes the total wall heat flux into mechanistic contributions associated with different physical processes. The most commonly used formulation is the RPI model of Kurul and Podowski [3], which partitions the heat flux into convective, quenching, and evaporative components. Since its introduction, many alternative HFP formulations have been proposed [4, 5, 6].

All HFP models require closure through sub-models that describe the underlying bubble dynamics [7]. These include empirical or semi-empirical expressions for bubble departure diameter and frequency, nucleation site density, and bubble wait and growth times. Each sub-model introduces its own assumptions, coefficients, and calibration range, and the interaction of multiple sub-models within different HFP formulations leads to substantial variability in predictive performance across operating conditions. This variability, together with the lack of universally reliable closures, remains a central challenge in the development of generalisable boiling models for Eulerian-Eulerian CFD simulations [8, 9, 10].

In our recent work we systematically investigated thousands of HFP and bubble parameter model combinations against a broad database of subcooled flow boiling experiments [11, 12]. It was shown that for each individual case it is always possible to find a model combination that matches the experimentally measured heat fluxes closely. However, there is no single universal model combination that can provide reasonable heat flux predictions across a wide range of flow conditions. In fact, even moderate changes in flow parameters, such as the pressure, can degrade the accuracy of a given model configuration. This is due to the limited ranges of validity of the models, often derived from a limited number of experimental measurements, the complex interdependencies of the various sub-models involved, and epistemic uncertainties in the models. The limited predictive accuracy motivates the need for an improved modelling framework that takes advantage of the capabilities of various existing models and applies them adaptively depending on operating conditions.

In this study, we propose a data-driven mixture-of-experts (MoE) approach for boiling model development. The concept of MoE originates from the work of Jacobs et al. [13], who introduced a neural architecture capable of decomposing complex mappings into simpler, specialised sub-models coordinated by a gating network. In this work, we extend this idea to the boiling problem where various HFP approaches and bubble dynamics models are treated as “experts”, while a gating function learns to weight their contributions in predicting the heat flux depending on flow conditions such as pressure, wall superheat, and subcooling. This strategy leverages the interpretability of mechanistic and semi-empirical models while exploiting machine learning to adaptively select or blend models based on experimental data. As a proof of concept, we employ a zero-dimensional modelling approach and a comprehensive database of subcooled flow boiling cases to examine whether it is possible to: (i) leverage the mixture-of-experts approach to improve global predictive performance of existing models; (ii) understand which boiling models the MoE relies on in different operating regimes; and (iii) provide a new paradigm of boiling modelling that integrates empirical and physics-based models with the adaptability of machine learning techniques.

The rest of the paper is structured as follows: Section 2 outlines the experimental datasets used for training and testing; Section 3 presents the methodology; Section 4 presents the results and discussion, with conclusions in Section 5.

2. EXPERIMENTAL DATASETS

The experimental data used for training and testing the proposed approach are based on subcooled flow boiling measurements from Phillips [14], Richenderfer et al. [15] and Su et al. [16]. These datasets are summarised in Table 1, with Fig. 1 illustrating the distribution of cases in the space of key flow conditions: pressure, subcooling, and bulk velocity.

The combined dataset was partitioned into three subsets, denoted as D_1 , D_2 , and D_3 , corresponding to the training, interpolation-testing, and extrapolation-testing domains, respectively.

Dataset D_1 comprises a subset of the measurements from Phillips [14], who acquired high-resolution boiling curves under subcooled flow conditions up to wall heat fluxes of approximately 1.6 MW/m^2 . These experiments

Table 1 Summary of operating conditions and dataset sizes for the experimental data used for training and testing the mixture-of-experts model. P : pressure, u : bulk velocity, ΔT_{sub} : subcooling, D_h : hydraulic diameter, q'' : total heat flux.

Dataset	P [bar]	u [m/s]	ΔT_{sub} [K]	D_h [m]	q'' [kW/m ²]	Cases	Points	Refs.
D_1 : training	1.05–2	0.16–1.30	5–15	0.010	87.7–1,601	27	252	[14]
D_2 : testing	1.05–2	0.16–1.30	5–15	0.010	3.20–1,605	27	250	[14]
D_3 : testing	1	0.5–2	5–25	0.015	98.4–5,367	12	255	[15, 16, 17]

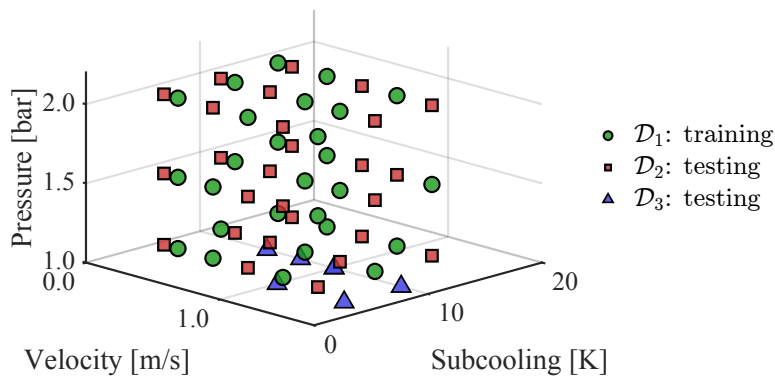


Fig. 1 Distribution of training and testing cases in flow condition space.

were deliberately operated *below* the onset of two-phase instabilities and well below the critical heat flux (CHF), and therefore provide data strictly within the nucleate boiling regime. Dataset D_1 is used for training the MoE, including validation during network optimisation.

Dataset D_2 contains the remaining Phillips cases not used for training, providing an independent interpolation test within the same operating range as D_1 . As all Phillips curves remain in subcooled nucleate boiling, D_2 evaluates the MoE’s capability to interpolate between known flow conditions without encountering regime-transition effects.

Dataset D_3 , on the other hand, consists of boiling curves from the Richenderfer [15] and Su [16] datasets. In contrast to the Phillips data, the Richenderfer experiments were designed to extend each boiling curve up to the critical heat flux. As a result, several cases in D_3 approach the upper end of the nucleate boiling regime and include strong pre-CHF effects not represented in the point-wise HFP models used here. Dataset D_3 therefore constitutes a stringent extrapolation test, evaluating the generalisation of the MoE to unseen data from independent facilities and into a regime closer to CHF.

3. METHODOLOGY

Fig. 2 illustrates the proposed mixture-of-experts approach. The total heat flux q'' is predicted as a weighted sum of various ‘experts’, where the weights are determined by a fully connected neural network with non-dimensionalised inputs, $\mathbf{x}$. The following sub-sections will explain the various components represented in Fig. 2.

3.1 Experts: boiling models In the zero-dimensional, point-averaged boiling framework, the wall heat flux is evaluated at a point representative of the surface-averaged conditions of the heated surface, eliminating the need to solve spatially-resolved transport equations. The total wall heat flux is expressed as an explicit function of the wall superheat, $\Delta T_w = T_{\text{wall}} - T_{\text{sat}}$, the flow conditions, and thermophysical properties of the fluid and surface. This approach provides a computationally efficient means to isolate and evaluate boiling heat transfer models, independent of uncertainties arising from other closures in Eulerian-Eulerian CFD simulations.

Mathematically, the total heat flux can be represented as a function of a chosen heat flux partitioning (HFP)

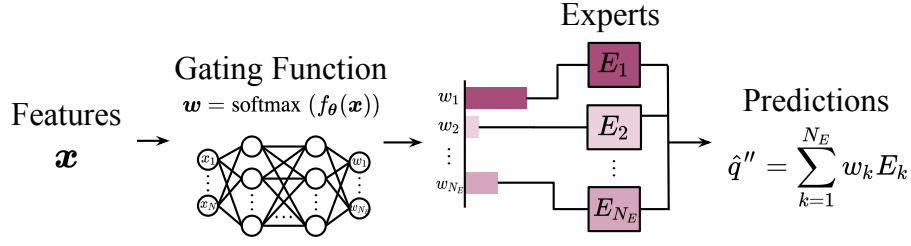


Fig. 2 The flow diagram of the proposed mixture-of-experts approach.

framework, $\mathcal{F}_{\text{HFP}}$, which in turn depends on several bubble dynamics parameters sub-models:

$$q''(\Delta T_w) = \mathcal{F}_{\text{HFP}}(D_d, f, \bar{N}_b, t_w, t_g, \text{Nu}, \varphi), \quad (1)$$

where D_d is the bubble departure diameter, f is the departure frequency, $\bar{N}_b$ is the nucleation site density, t_g is the bubble growth time, t_w is the bubble wait time, Nu is the Nusselt number, and φ represents other flow and thermophysical parameters, such as pressure, subcooling, geometric parameters such as hydraulic diameter, and thermophysical properties of the heating surface and the working fluid. Each HFP formulation prescribes a specific decomposition of q'' into constituent mechanisms. For example, the widely used RPI model expresses the total heat flux as

$$\mathcal{F}_{\text{RPI}} = q''_{\text{convection}} + q''_{\text{evaporation}} + q''_{\text{quenching}}, \quad (2)$$

$$q''_{\text{convection}} = h_c \left(1 - \frac{\pi}{4} D_d^2 K \bar{N}_b \right) (\Delta T_w + \Delta T_{\text{sub}}), \quad (3)$$

$$q''_{\text{evaporation}} = \frac{\pi}{6} \rho_v h_{lv} D_d^3 f \bar{N}_b, \quad (4)$$

$$q''_{\text{quenching}} = t_w f \frac{\pi}{4} D_d^2 K \bar{N}_b \frac{2k_l(\Delta T_w + \Delta T_{\text{sub}})}{\sqrt{\pi t_w k_l / \rho_l c_{p,l}}}, \quad (5)$$

where h_c is the single-phase convection heat transfer coefficient (as a function of the Nusselt number), K is the bubble growth constant, ρ_v is the vapour density, h_{lv} is latent heat of vaporisation, k_l is the liquid thermal conductivity, ρ_l is the liquid density, and $c_{p,l}$ is the liquid specific heat capacity.

In this work, we consider a large library of boiling models constructed through combinatorial variations of two heat flux partitioning frameworks and various sub-models for bubble dynamics parameters, fully detailed in [12]. Each unique combination of one HFP framework and a set of sub-models constitutes a distinct boiling model configuration, denoted as an expert in the present mixture-of-experts (MoE) formulation. These experts collectively span a wide range of mechanistic hypotheses and empirical assumptions underlying wall heat transfer during subcooled flow boiling.

Mathematically, the experts can be expressed as

$$E_k = \left\{ \mathcal{F}_{\text{HFP}}^{(k)} \left(D_d^{(k)}, f^{(k)}, \bar{N}_b^{(k)}, t_g^{(k)}, t_w^{(k)}, \text{Nu}^{(k)} \right) \right\}, \quad (6)$$

where each expert $k \in \mathbb{R}^{N_E}$ corresponds to a distinct selection of sub-models, with N_E representing the total number of experts.

Initially, over 16,000 distinct model configurations are available as potential experts based on our implementation in XBoil [12]. However, a substantial fraction of these configurations produce highly inaccurate heat flux predictions. To ensure consistency and reduce redundancy, the expert pool is filtered through a systematic process involving three main steps. First, configurations using the Kim and Kim HFP formulation are excluded¹, as this model was originally developed for pool boiling and was found to perform poorly under flow-boiling conditions. Second, for each training case, the remaining configurations are ranked according to their mean squared error against experimental boiling curves, and the top 100 are retained. Finally, to avoid near-duplicate

¹In total three different heat flux partitioning models have been implemented in XBoil.

experts, configurations exhibiting highly correlated predictions (Pearson correlation coefficient > 0.999) are deduplicated, retaining only the representative model with the lowest error. This process leads to a diverse set of 215 experts, which are listed in Table 2.

Table 2 Summary of sub-models represented in the final global expert pool (215 experts).

Category	Models included
HFP	RPI [3], MITB [5, 18]
Departure diameter	Bovard [19], Cole [20], Colombo & Fairweather [21], Fritz [22], MIT [18], Nam [23], Phan [24], Ruckenstein [25], Tolubinsky & Kostanchuk [26], Van Stralen & Zijil [27]
Departure frequency	Cole [28], Peebles & Garber [29], Sakashita & Ono [30], Stephan [31], Zuber [32]
Nucleation site density	Hibiki & Ishii [33], Hibiki–Ishii–MIT [18], Lemmert & Chawla [34], Li [35], Wang & Dhir [36], Yang [37]
Growth time	MIT [18], Lee et al. [38], Van Stralen et al. [27]
Wait time	Van Stralen et al. [27], MIT [18]
Nusselt number correlation	Churchill & Chu [39], Gnielinski [40]

Fig. 3a illustrates five randomly selected experts from the filtered pool, showing the wide variability in predictive behaviour across different model combinations. While some experts exhibit reasonable agreement with experimental data, others substantially over- or under-predict the heat flux, reflecting the limited generalisability of individual HFP formulations. In contrast, Fig. 3b presents the best-performing expert for each case, highlighting that although individual models can achieve high accuracy locally, no single expert performs consistently well across all operating conditions. This observation motivates the use of a data-driven mixture-of-experts framework to combine the strengths of multiple models adaptively.

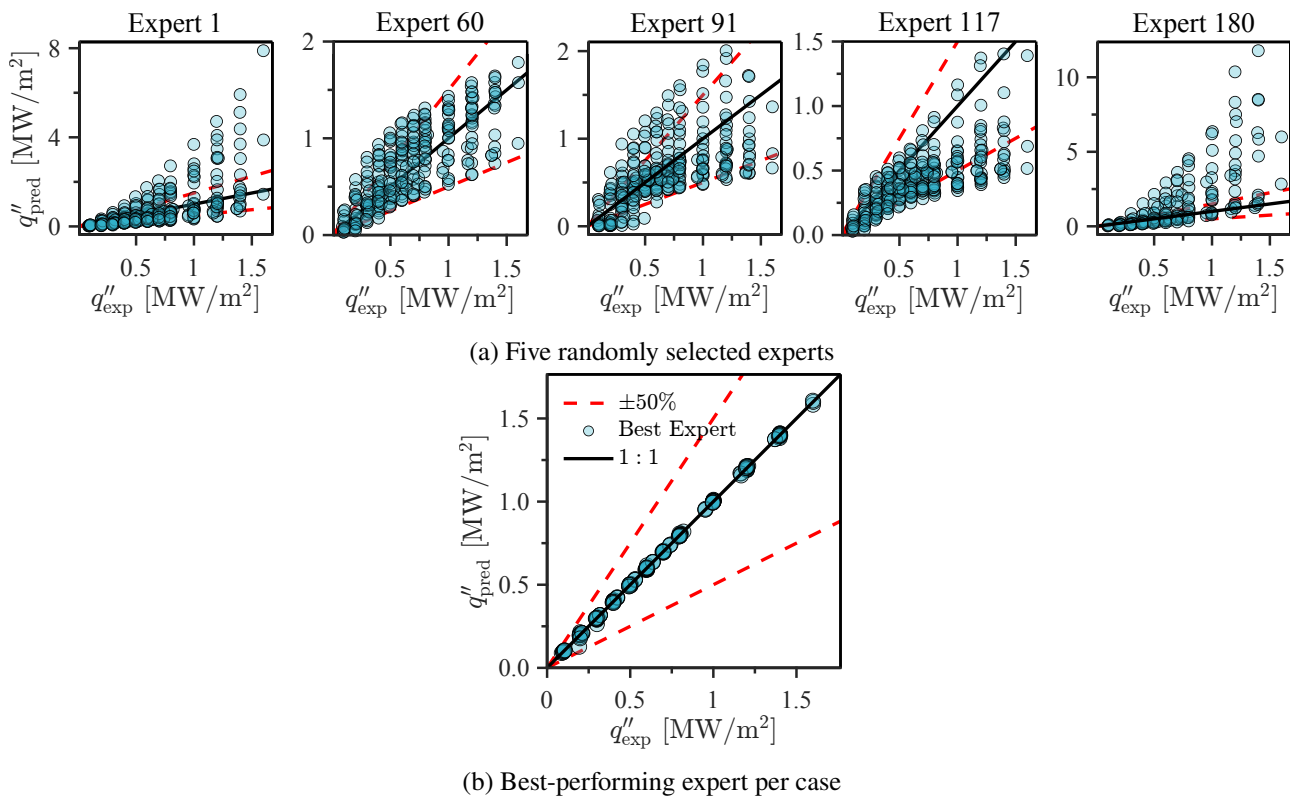


Fig. 3 Comparison of predicted versus experimental heat fluxes.

3.2 Mixture-of-expert architecture and training strategy Within the MoE framework, a data-driven gating function learns to assign adaptive weights to the experts depending on the operating conditions, enabling the final prediction to integrate the strengths of multiple HFP formulations.

Within the MoE framework, a data-driven gating function learns to assign adaptive weights to the experts depending on the operating conditions, enabling the final prediction to integrate the strengths of multiple HFP formulations. Mathematically, the predicted non-dimensional heat flux $\hat{q}''$ is expressed as a weighted sum of the N_E expert predictions,

$$\hat{q}'' = \sum_{k=1}^{N_E} w_k(\mathbf{x}) q_k''(E_k, \varphi), \quad \text{with} \quad \sum_{k=1}^{N_E} w_k = 1, \quad w_k \geq 0, \quad (7)$$

where q_k'' is the prediction from expert k for a given set of flow and thermophysical parameters φ , and $w_k(\mathbf{x})$ is the corresponding gating weight. The weights are obtained from a neural network (the gating function) that takes as input a normalised feature vector

$$\mathbf{x} = [P/P_{\text{crit}}, Ja_{\text{sub}}, Re, Pr, We, \Delta T_w/T_{\text{sat}}, Ja_{\text{sup}}], \quad (8)$$

where P is pressure, P_{crit} is the fluid critical pressure, Ja , Re , Pr , and We are the Jakob, Reynolds, Prandtl, and Weber numbers. The features $\mathbf{x}$ are selected to encapsulate the operating and thermophysical parameters φ . The system pressure normalised by the critical pressure to include pressure effects, the Jakob number $Ja = c_{p,l} \Delta T_{\text{sub}} / h_{lv}$ quantifies the available sensible heat relative to latent heat and accounting for the subcooling, and the Reynolds, Prandtl, and Weber numbers (Re , Pr , We) describe flow inertia, thermal diffusivity, and surface tension effects, respectively. The wall superheat ratio $\Delta T_w / T_{\text{sat}}$ and the local Jakob number $Ja_{\text{sup}} = c_{p,l} \Delta T_w / h_{lv}$ further account for the degree of wall heating and its coupling to bubble nucleation and growth dynamics. All features are normalised using the mean and standard deviation of the training data to ensure balanced scaling and stable network optimisation.

The gating network outputs an unnormalised vector of logits $\mathbf{z}(\mathbf{x}) \in \mathbb{R}^{N_E}$, which are converted to probabilities using a softmax operation,

$$w_k(\mathbf{x}) = \frac{\exp(z_k)}{\sum_{j=1}^{N_E} \exp(z_j)}. \quad (9)$$

This ensures that all gating weights are positive and sum to unity.

The network parameters $\boldsymbol{\theta}$ are optimised by minimising the mean Huber loss $\mathcal{L}$ between the predicted and target non-dimensional heat fluxes over all training samples i ,

$$\mathcal{L}(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^N \begin{cases} \frac{1}{2} \varepsilon_i^2, & |\varepsilon_i| \leq \delta, \\ \delta(|\varepsilon_i| - \frac{1}{2}\delta), & |\varepsilon_i| > \delta, \end{cases} + \lambda \|\boldsymbol{\theta}\|_2^2, \quad (10)$$

where $\varepsilon_i = \hat{q}_i'' - q_{\text{exp},i}''$ is the prediction error, $\delta = 1$ is the Huber threshold, and λ is the weight-decay coefficient controlling L2 regularisation. The optimisation is performed using the Adam algorithm with mini-batch training.

To identify the optimal network architecture and training configuration, a random hyperparameter search was conducted across a broad space of learning rate, weight decay, batch size, number of layers, neurons per layer, and activation function. Each trial was trained using an 90/10 split of the training data for model and validation loss monitoring. The hyperparameter combinations yielding the lowest validation loss were selected for final training. Fig. 4 summarises the explored design space and highlights the best-performing configuration.

4. RESULTS AND DISCUSSIONS

Fig. 5 compares the mixture-of-experts (MoE) predictions against the experimental heat fluxes for the three datasets described in Section 2. For the training domain (D_1), which consists of nucleate-boiling data from Phillips acquired well below the critical heat flux (CHF), the MoE achieves very close agreement with the

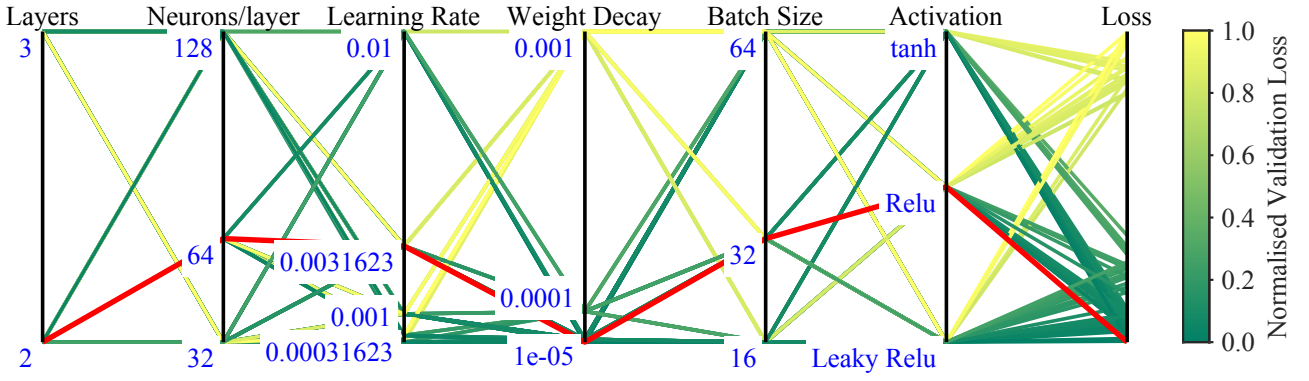


Fig. 4 Parallel plot illustrating the normalised validation loss from the random hyperparameter search for neural network training. The tuned parameters for the final neural network is highlighted in red.

experiments, with nearly all points falling within the $\pm 50\%$ bounds. This indicates that the gating function successfully learns to blend the experts in a physically consistent manner within the regime represented in the training data. The interpolation-testing dataset (D_2), comprising the remaining Phillips cases, shows similarly good agreement: the predictions remain well correlated with experiments and demonstrate that the MoE generalises reliably to intermediate flow conditions within the same nucleate-boiling regime. This is further reflected in the accurate predictions of the boiling curves shown in Fig. 6 (a)-(b).

The extrapolation-testing dataset (D_3), on the other hand, contains independent measurements from D_1 and D_2 datasets as discussed in Section 2. The D_3 dataset boiling curves were acquired up to the critical heat flux. Consequently, the upper portions of these boiling curves are significantly more challenging to reproduce due to the complex physical mechanisms near CHF. While the model follows the experimental trend reasonably well at lower wall superheats, where the behaviour remains consistent with nucleate boiling, it increasingly underpredicts the heat flux as the curves approach CHF, as illustrated in Fig. 6 (c). This degradation is expected: the MoE is trained exclusively on sub-CHF data and has no representation of the physics governing the transition to departure from nucleate boiling (DNB). Nevertheless, the results demonstrate that the MoE can reproduce the qualitative evolution of the boiling curve in the nucleate-boiling regime, while its limitations become clear as the operating conditions move beyond the domain of the training data.

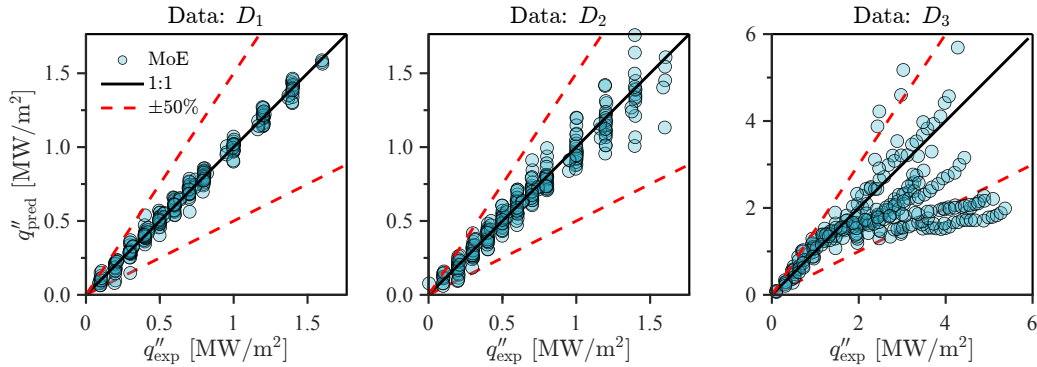


Fig. 5 Scatter plots comparing the experimental heat fluxes against the mixture-of-experts model predictions for the three datasets in Table 1.

Fig. 7 presents a comparison of the mean absolute percentage error (MAPE) for all individual experts (existing configurations, without data-driven modelling) and the mixture-of-experts model across the three datasets. The performance of the existing model configurations is highly variable, with errors spanning several orders of magnitude depending on the particular combination of HFP and bubble dynamics sub-models. This variability is evident not only across the global ensemble but also for individual experts when evaluated on the different datasets. The MoE model, although not universally superior, achieves notably lower average errors than the mean of the individual experts across all three datasets, with MAPEs of 4.9%, 19.3%, and 23.0% for D_1 , D_2 , and D_3 , respectively. The relatively low training error indicates effective fitting of the available data,

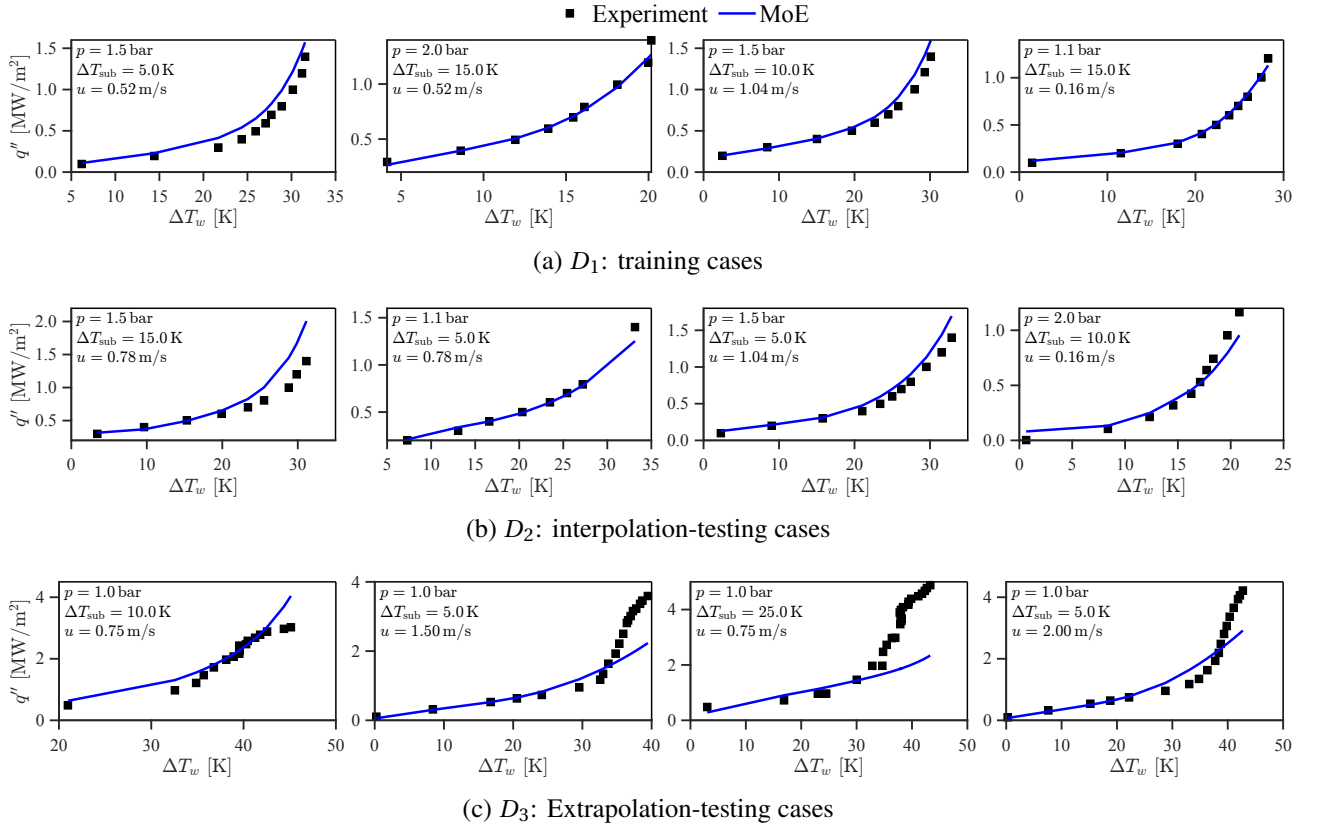


Fig. 6 Representative boiling curves predicted by the MoE model for the three datasets.

while the increased errors for D_2 and particularly for the extrapolation set D_3 suggest that the model partially inherits the limited generality of its constituent experts. Nonetheless, the MoE demonstrates a moderate degree of extrapolative capability, achieving comparable accuracy to the best-performing physical models without explicit re-tuning, and highlighting both the promise and the current limitations of data-driven weighting in capturing boiling behaviour beyond the training regime.

Fig. 8 shows the mean gating weights $\bar{w}_k$ assigned by the MoE model to each expert across the three datasets. Only a small fraction of the total expert pool receives non-negligible weights ($\bar{w}_k \geq 0.01$), indicating that the MoE effectively prunes redundant or low-utility experts, relying instead on a limited subset of configurations that best predict the heat fluxes. The distributions for the training (D_1) and interpolation-testing (D_2) datasets are closely aligned, reflecting the similar flow and operating conditions between the two. This consistency suggests that the gating network generalises smoothly within the same regime, assigning comparable importance to the same experts. In contrast, for the extrapolation-testing dataset D_3 , the gating weights are several orders of magnitude smaller and more dispersed. This behaviour reflects increased model uncertainty when exposed to conditions beyond the training domain, highlighting a lack of generalisability.

Fig. 9 compares the mean gating weights assigned by the MoE model to each expert against their individual prediction errors across the three datasets. A general negative correlation is observed, indicating that the MoE tends to assign higher weights to more accurate experts. This relationship is strongest in the training (D_1) and interpolation-testing (D_2) datasets, where the model has learned to prioritise configurations that produce lower errors within the operating regime seen during training. For the extrapolation dataset (D_3), however, the correlation weakens, and the absolute weights are several orders of magnitude smaller, as also shown in Fig. 8. This behaviour suggests that the gating function is less confident when encountering unseen flow and operating conditions, distributing smaller and more uniform weights across the expert pool.

Understanding which input features most strongly influence the model's predictions is essential for interpreting how the MoE framework allocates weight across operating conditions. To this end, a feature ablation analysis was performed, where each input feature was sequentially removed and the resulting change in prediction error (ΔMAPE) was evaluated. This approach provides a straightforward means of quantifying the relative

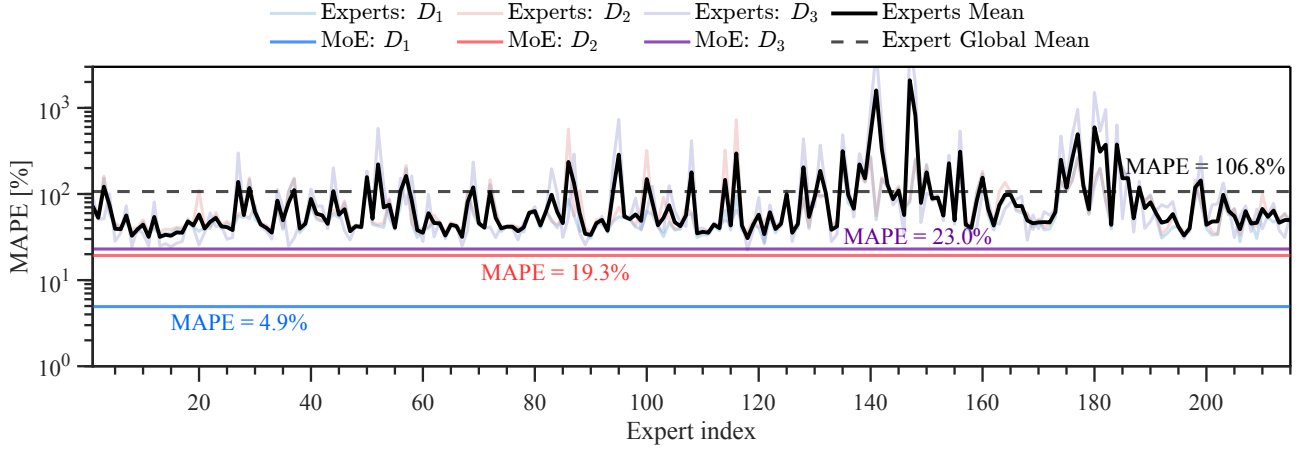


Fig. 7 Comparison of per-expert and mixture-of-experts (MoE) performance across datasets D_1 – D_3 . Faint lines show the mean absolute percentage error (MAPE) of individual experts evaluated on each dataset, while the thick black line denotes the mean MAPE of each expert across all three datasets. Solid horizontal lines indicate the MoE errors for training (D_1), testing (D_2), and extrapolation (D_3) domains.

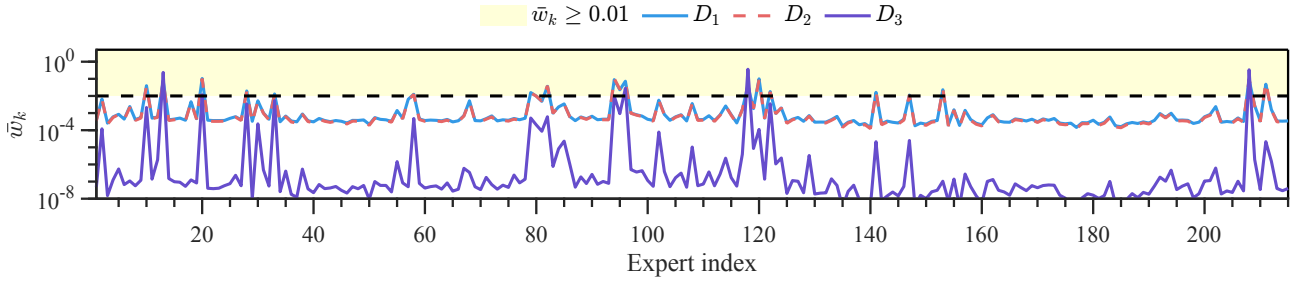


Fig. 8 Comparison of the mean gating weights $\bar{w}_k$ assigned by the MoE model to each expert across the datasets. The shaded region denotes experts with mean weights above 0.01, indicating those contributing significantly to the overall prediction.

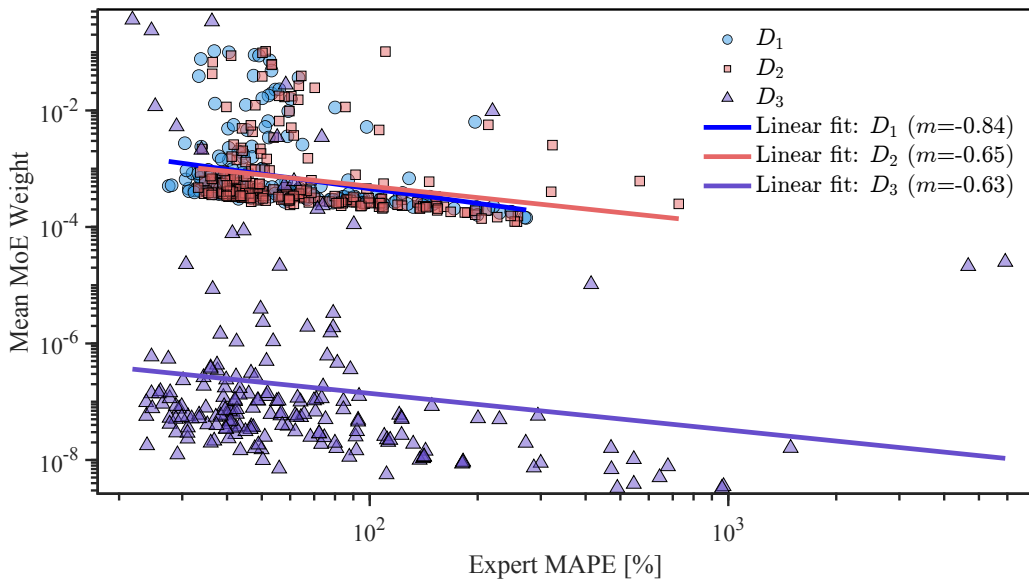


Fig. 9 Correlation between the mean gating weights assigned by the MoE model and the individual expert prediction errors across the datasets.

importance of different features, though it does not account for correlations or non-linear interactions between them and thus reflects the model’s learned sensitivities rather than causal physical relationships. As shown in Fig. 10, the reduced pressure, p/p_{crit} , is the most influential feature, highlighting the dependence of boiling heat transfer on pressure and its influence on thermophysical properties such as latent heat and surface tension. The Prandtl number (Pr), Weber number (We) and subcooling Jakob number (Ja_{sub}) also show high importance, indicating their role in capturing the balance between convection and phase-change effects. In contrast, the Reynolds number (Re) and wall superheat ratio ($\Delta T_w/T_{\text{sat}}$) have smaller impacts. The limited sensitivity to Re likely reflects the narrow range of velocities and hydraulic diameters across the datasets. The low apparent importance of the wall superheat ratio is more nuanced: although it is the key variable governing the boiling curve within each expert, the gating function primarily determines which expert dominates under different thermophysical and flow conditions, rather than directly shaping the heat flux–superheat relationship. Consequently, the gating network relies more on global descriptors such as pressure and subcooling, while the dependence on wall superheat is embedded within the expert models themselves.

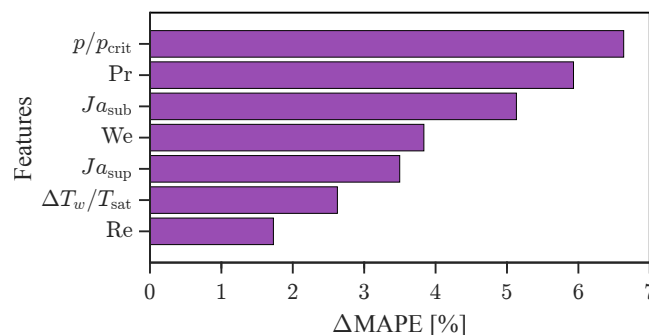


Fig. 10 Feature importance analysis using a leave-one-feature-out ablation study. Each bar shows the increase in mean absolute percentage error on the test dataset D_2 when the corresponding input feature is removed. Features are sorted in descending order of impact.

5. CONCLUSIONS

This paper presented our on-going work on a data-driven mixture-of-experts (MoE) framework for modelling subcooled flow boiling by combining the predictive capabilities of existing heat flux partitioning (HFP) models. Within this approach, each combination of HFP and bubble dynamics sub-models was treated as an individual expert, while a neural gating network learned to assign adaptive weights based on flow and thermophysical conditions.

The MoE model demonstrated improved predictive performance relative to the constituent experts, particularly in interpolation regimes, while maintaining reasonable accuracy when extrapolated to independent datasets. Only a small subset of the 215 available experts received significant weights, indicating that the model effectively identified and relied on the most physically relevant configurations. The observed correlation between expert accuracy and assigned weight further confirmed that the gating function learned to prioritise well-performing models. Feature ablation analysis showed that reduced pressure and subcooling parameters exert the greatest influence on the gating behaviour, whereas features such as the Reynolds number and wall superheat ratio played smaller roles. This is consistent with the relatively narrow span of pressures represented in the available experimental datasets, which limits the model’s ability to learn broader pressure dependencies. Extending the framework to include datasets covering a wider pressure envelope will therefore be an important focus of future work.

While the present study demonstrates the feasibility of the MoE framework for boiling heat transfer predictions as an early proof-of-concept, several challenges remain. The current formulation is restricted to a zero-dimensional representation of wall boiling and relies on a relatively narrow experimental database. Future work will therefore focus on extending the framework to Eulerian-Eulerian CFD simulations, and expanding the training database to include a broader span of operating conditions. Incorporating formal uncertainty quantification to account for both experimental and model-form variability, and embedding additional physical

constraints to improve robustness when extrapolating beyond the training datasets will also be essential for developing a more general and transferable modelling capability.

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NOMENCLATURE

D_d	bubble departure diameter	(m)	z_k	gating-network logit associated with expert k	(-)
E_k	k^{th} expert model configuration	(-)	δ	Huber-loss threshold parameter	(-)
f	bubble departure frequency	(s ⁻¹)	ΔT_{sub}	liquid subcooling	(K)
$\mathcal{F}_{\text{HFP}}$	heat flux partitioning function	(-)	ΔT_w	wall superheat	(K)
Ja_{sub}	subcooling Jakob number	(-)	ϵ_i	prediction error for sample i	(-)
Ja_{sup}	wall-superheat Jakob number	(-)	ϕ	additional flow and thermophysical parameters in the HFP formulation	(-)
$\bar{N}_b$	nucleation site density	(m ⁻²)	$\hat{q}''$	MoE predicted heat flux	(-)
N_E	total number of experts	(-)	λ	weight-decay coefficient	(-)
P_{crit}	critical pressure	(Pa)	θ	neural-network trainable parameters	(-)
t_g	bubble growth time	(s)			
t_w	bubble wait time	(s)			
w_k	gating weight for expert k	(-)			
$\mathbf{x}$	input feature vector to the gating network	(-)			

REFERENCES

- [1] Wu, M., Zhang, J., Gui, N., Zou, Q., Yang, X., Tu, J., Jiang, S. and Liu, Z., “Advances in the modeling of multiphase flows and their application in nuclear engineering—A review”, *Experimental and Computational Multiphase Flow*, 6(4), pp. 287–352 (2024).
- [2] Prosperetti, A. and Tryggvason, G., *Computational methods for multiphase flow*, Cambridge university press (2009).
- [3] Kurul, N. and Podowski, M. Z., “Multidimensional effects in forced convection subcooled boiling”, *Proceeding of International Heat Transfer Conference 9*, Begellhouse (1990).
- [4] Warrier, G. R. and Dhira, V. K., “Heat Transfer and Wall Heat Flux Partitioning During Subcooled Flow Nucleate Boiling—A Review”, *Journal of Heat Transfer*, 128(12), p. 1243–1256 (2006).
- [5] Gilman, L. and Baglietto, E., “A self-consistent, physics-based boiling heat transfer modeling framework for use in computational fluid dynamics”, *International Journal of Multiphase Flow*, 95, pp. 35–53 (2017).
- [6] Zhou, P., Hua, S., Gao, C., Sun, D. and Huang, R., “A mechanistic model for wall heat flux partitioning based on bubble dynamics during subcooled flow boiling”, *International Journal of Heat and Mass Transfer*, 174, p. 121295 (2021).
- [7] Mohanty, R. L. and Das, M. K., “A critical review on bubble dynamics parameters influencing boiling heat transfer”, *Renewable and Sustainable Energy Reviews*, 78, p. 466–494 (2017).
- [8] Pham, M., Bois, G., Francois, F. and Baglietto, E., “Assessment of State-of-The-Art Multiphase CFD Modeling for Subcooled Flow Boiling in Reactor Applications”, *Nuclear Engineering and Design*, 411, p. 112379 (2023).
- [9] Bois, G., Fillion, P., François, F., Burlot, A., Ali, A. B. H., Khaware, A., Sanyal, J., Rehm, M., Farges, B., Vinauger, F. et al., “Benchmark DEBORA: Assessment of MCFD compared to high-pressure boiling pipe flow measurements”, *International Journal of Multiphase Flow*, 179, p. 104920 (2024).
- [10] Colombo, M., Rzehak, R., Fairweather, M., Liao, Y. and Lucas, D., “Benchmarking of computational fluid dynamic models for bubbly flows”, *Nuclear Engineering and Design*, 375, p. 111075 (2021).
- [11] Bidar, O. and Colombo, M., “Assessment of heat flux partitioning approaches for the prediction of subcooled flow boiling”, *Proceedings of 21st International Topical Meeting on Nuclear Reactor Thermal Hydraulics* (2025).
- [12] Bidar, O. and Colombo, M., “Evaluations of boiling heat flux partitioning and bubble parameter models using a 0-D framework”, Preprint (2025).
- [13] Jacobs, R. A., Jordan, M. I., Nowlan, S. J. and Hinton, G. E., “Adaptive Mixtures of Local Experts”, *Neural Computation*, 3(1), pp. 79–87 (1991).
- [14] Phillips, B. A., *Experimental investigation of subcooled flow boiling using synchronized high speed video, infrared thermography, and particle image velocimetry*, Ph.D. thesis, Massachusetts Institute of Technology (2014).

- [15] Richenderfer, A., Kossolapov, A., Seong, J. H., Saccone, G., Demarly, E., Kommajosyula, R., Baglietto, E., Buongiorno, J. and Bucci, M., "Investigation of subcooled flow boiling and CHF using high-resolution diagnostics", *Experimental Thermal and Fluid Science*, 99, pp. 35–58 (2018).
- [16] Su, G.-Y., Wang, C., Zhang, L., Seong, J. H., Kommajosyula, R., Phillips, B. and Bucci, M., "Investigation of flow boiling heat transfer and boiling crisis on a rough surface using infrared thermometry", *International Journal of Heat and Mass Transfer*, 160, p. 120134 (2020).
- [17] Richenderfer, A. J., *Experimental study of heat flux partitioning in pressurized subcooled flow boiling*, Ph.D. thesis, Massachusetts Institute of Technology (2018).
- [18] Kommajosyula, R., *Development and assessment of a physics-based model for subcooled flow boiling with application to CFD*, Ph.d. thesis, Massachusetts Institute of Technology (2020).
- [19] Bovard, S., Asadinia, H., Hosseini, G. and Alavi Fazel, S. A., "Investigation and experimental analysis of the bubble departure diameter in pure liquids on horizontal cylindrical heater", *Heat and Mass Transfer*, 53(4) (2016).
- [20] Cole, R., "A photographic study of pool boiling in the region of the critical heat flux", *AIChE Journal*, 6(4), pp. 533–538 (1960).
- [21] Colombo, M. and Fairweather, M., "Prediction of bubble departure in forced convection boiling: A mechanistic model", *International Journal of Heat and Mass Transfer*, 85, pp. 135–146 (2015).
- [22] Friz, W., "Maximum volume of vapor bubbles", *Physikalische Zeitschrift*, 36, pp. 379–354 (1935).
- [23] Nam, Y., Aktinol, E., Dhir, V. K. and Ju, Y. S., "Single bubble dynamics on a superhydrophilic surface with artificial nucleation sites", *International Journal of Heat and Mass Transfer*, 54(7–8), pp. 1572–1577 (2011).
- [24] Phan, H. T., Caney, N., Marty, P., Colasson, S. and Gavillet, J., "A model to predict the effect of contact angle on the bubble departure diameter during heterogeneous boiling", *International Communications in Heat and Mass Transfer*, 37(8), pp. 964–969 (2010).
- [25] Ruckenstein, R., "Recent trends in boiling heat transfer research", *Applied Mechanics Reviews*, 17, pp. 663–672 (1964).
- [26] Tolubinsky, V. I. and Kostanchuk, D., "Vapour bubbles growth rate and heat transfer intensity at subcooled water boiling", *Proceeding of International Heat Transfer Conference 4*, Begellhouse (1970).
- [27] van Stralen, S., Sohal, M., Cole, R. and Sluyter, W., "Bubble growth rates in pure and binary systems: Combined effect of relaxation and evaporation microlayers", *International Journal of Heat and Mass Transfer*, 18(3), p. 453–467 (1975).
- [28] Cole, R., "Bubble frequencies and departure volumes at subatmospheric pressures", *AIChE Journal*, 13(4), pp. 779–783 (1967).
- [29] Peebles, F. N. and Garber, H. J., "Studies on the motion of gas bubbles in liquids", *Chemical Engineering Progress*, 49(2), pp. 88–97 (1953).
- [30] Sakashita, H. and Ono, A., "Boiling behaviors and critical heat flux on a horizontal plate in saturated pool boiling of water at high pressures", *International Journal of Heat and Mass Transfer*, 52(3-4), pp. 744–750 (2009).
- [31] Stephan, K., "Enhancement of Heat Transfer During Boiling", In *Heat Transfer in Condensation and Boiling*, Springer, pp. 292–301 (1992).
- [32] Zuber, N., "Nucleate boiling. The region of isolated bubbles and the similarity with natural convection", *International Journal of Heat and Mass Transfer*, 6(1), pp. 53–78 (1963).
- [33] Hibiki, T. and Ishii, M., "Active nucleation site density in boiling systems", *International Journal of Heat and Mass Transfer*, 46(14), pp. 2587–2601 (2003).
- [34] Lemmert, M. and Chawla, L. M., "Influence of flow velocity on surface boiling heat transfer coefficient", *International Heat Transfer Conference Proceedings*, published in 1977 (1974).
- [35] Li, Q., Jiao, Y., Avramova, M., Chen, P., Yu, J., Chen, J. and Hou, J., "Development, verification and application of a new model for active nucleation site density in boiling systems", *Nuclear Engineering and Design*, 328, pp. 1–9 (2018).
- [36] Wang, C. H. and Dhir, V. K., "Effect of surface wettability on active nucleation site density during pool boiling of water on a vertical surface", *ASME Journal of Heat Transfer*, 115, pp. 659–669 (1993).
- [37] Yang, L. X., Guo, A. and Liu, D., "Experimental Investigation of Subcooled Vertical Upward Flow Boiling in a Narrow Rectangular Channel", *Experimental Heat Transfer*, 29(2), pp. 221–243 (2014).
- [38] Lee, H. C., Oh, B. D., Bae, S. W. and Kim, M. H., "Single bubble growth in saturated pool boiling on a constant wall temperature surface", *International Journal of Multiphase Flow*, 29(12), p. 1857–1874 (2003).
- [39] Churchill, S. W. and Chu, H. H. S., "Correlating equations for laminar and turbulent free convection from a vertical plate", *International Journal of Heat and Mass Transfer*, 18(11), pp. 1323–1329 (1975).
- [40] Gnielinski, V., "New equations for heat and mass transfer in turbulent pipe and channel flow", *International Chemical Engineering*, 16(2), pp. 359–368 (1976).